

CS 383
HW 3

Due in class Wednesday, October 2.

This one should be typed. Several of the questions ask for either a DFA or a regular expression. It is fine to attach a page to your solutions on which you have drawn the DFAs.

1. Either prove or give an example that disproves: For any regular expressions E and F , $(E+F)^* = E^*+F^*$.
2. Show that the language of strings of balanced parentheses (e.g. “((()))”) is not a regular language.
3. Give a careful proof that $\{a^n b^m c^n \mid n \geq 0, m \geq 0\}$ is not regular.
4. For each of the following languages, either prove that it is regular (by giving a regular expression or DFA for it) or use the Pumping Lemma to prove that it isn't regular.
 - a. The set of strings of 0's and 1's where the digits sum to 5, such as 110111 and 11111.
 - b. The set of strings of 0's and 1's where the digits sum to an even number.
 - c. The set of strings of 0's and 1's where the digits sum to a perfect square.
 - d. The set of strings of 0's and 1's such that in every prefix the number of 0's and the number of 1's never differ by more than 1. 011001 is such a string.
5. If L is a language and a is a symbol then L/a (called the *quotient* of L and a) is the set of strings w such that wa is in L . For example, if $L = \{a, aab, baa\}$ then $L/a = \{\epsilon, ba\}$. Show that if L is regular then L/a is also regular.
6. Suppose I give you a very complicated DFA for a language over the alphabet $\{0,1\}$. Give an algorithm for determining if the language accepted by that DFA is infinite. The algorithm doesn't need to be efficient, it just needs to be able to eventually give a definite “yes” or “no” answer. *Hint*: The Pumping lemma helps. *Another hint*: It isn't sufficient to treat the DFA like a directed graph and ask if it has a cycle.